

Positron production in collision of heavy nuclei

I.B. Khriplovich*, D.A. Solov'yev†

*Department of Physics, St. Petersburg State University,
Ul'ianovskaya 1, Petrodvorets, 198504 St. Petersburg, Russia*

(Dated: June 27, 2016)

We consider the electromagnetic production of positron in collision of heavy nuclei, with the simultaneously produced electron captured by one of the nuclei. This cross-section exceeds essentially the cross-section of e^+e^- production.

Positron production in collision of heavy nuclei (as well as the production of e^+e^- pair), was addressed in numerous papers (see, for instance, [1]-[3]). In the present note we reconsider the problem of the positron production in collision of heavy nuclei, and derive a simple estimate for the corresponding cross-section.

The diagram related to our problem is presented in Fig. 1. The bold lines in this diagram correspond to the propagation of non-relativistic heavy nuclei with initial momenta $\pm\bar{p}$, and final momenta $\bar{p}_{1,2}$. For the velocities of nuclei we assume $v = 0.1$ (or $v/c = 0.1$ in usual units). The wavy line in Fig. 1 refers to the virtual photon producing the pair e^+e^- . The produced electron is captured immediately by one of the nuclei, thus creating a single-electron ion; this ion by itself is also on mass shell. The four-momentum of the produced positron is $k_\mu = (k_0, \vec{k})$, $k_0 = \sqrt{k^2 + m^2}$, we will neglect here the interaction of positron with nuclei.

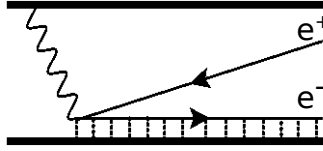


FIG. 1: Feynman diagram of positron production.

We start the analysis with rather obvious conservation law formulated as follows:

$$\int d\bar{p}_1 d\bar{p}_2 \frac{1}{(\bar{p} - \bar{p}_1)^4} \delta(\bar{p}_1 + \bar{p}_2 + \vec{k}) \delta(\bar{p}^2/M - \bar{p}_1^2/2M - \bar{p}_2^2/2M - k_0). \quad (1)$$

Here and below M is the mass of colliding nuclei, and one nucleus has captured the outgoing electron. The factor $1/(\bar{p} - \bar{p}_1)^4$ corresponds to the momentum transfer via the photon exchange between nuclei. Integral (1) is the only expression in this problem which depends on \bar{p}_1 and \bar{p}_2 . Therefore, we can integrate (1) freely over \bar{p}_1 and \bar{p}_2 .

Of course, the nucleus which captures the electron will be slightly more heavy than the another one. We neglect here this tiny difference. Now, integrating (1) over \bar{p}_2 , we arrive at

$$\int d\bar{p}_1 \frac{1}{(\bar{p} - \bar{p}_1)^4} \delta(\bar{p}^2/M - \bar{p}_1^2/M - k_0); \quad (2)$$

here we have omitted negligibly small term $\bar{p}_1 \vec{k}/M$ in the argument of delta-function.

Integral (2) is conveniently rewritten as follows:

$$\int \frac{d\vec{q}}{\vec{q}^4} \delta(2\vec{v}\vec{q} - k_0), \quad (3)$$

we have neglected here one more small term $-\vec{q}^2/M$. Let us split now vector \vec{q} into the part parallel to \vec{v} , and the part orthogonal to \vec{v} , i.e. into q_{\parallel} and \vec{q}_{\perp} . Then

$$\int_0^\infty \frac{2\pi q_{\perp} dq_{\perp}}{(\vec{q}_{\perp}^2 + q_{\parallel}^2)^2} = \frac{\pi}{q_{\parallel}^2}. \quad (4)$$

* khios231@mail.ru

† solov'yev.d@gmail.com

At last, we integrate over q_{\parallel} :

$$\pi \int_{-\infty}^{\infty} \frac{dq_{\parallel}}{q_{\parallel}^2} \delta(2vq_{\parallel} - k_0) = \pi \int_{-\infty}^{\infty} \frac{dx}{x^2} 2v \delta(x - k_0) = 2\pi v/k_0^2. \quad (5)$$

Let us address now the fermion loop arising in the amplitude squared, see Fig. 2. The upper part of this loop, corresponding to the positron propagation, is $\hat{k} - m$ (as mentioned, we neglect the positron rescattering). The lower part of this loop is a single-electron ion of small velocity $v \sim 0.1$, and we will neglect its velocity at all. Then, the only structure surviving here is

$$Sp(\hat{k} - m)\gamma_3\gamma_0\gamma_3 = 4k_0. \quad (6)$$

We assume here that the nuclei propagate along the z axis; this is the origin of γ_3 in the above formula.

The net result of formulas (1) – (6) is

$$\frac{2\pi v}{k_0^2} 4k_0 = \frac{8\pi v}{k_0}. \quad (7)$$

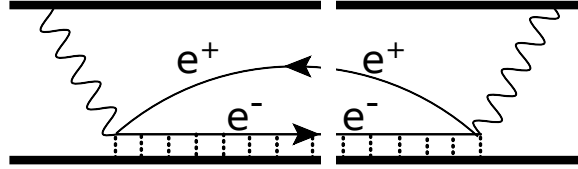


FIG. 2: The loop, corresponding to positron propagation.

Less obvious is the dependence of the discussed cross-section on the velocity v of the colliding nuclei. The common electromagnetic vertex of heavy nucleus is proportional to the velocity v , which gives rise to v^2 in the cross-section (see Fig. 2). Then, the flux density of nuclei is proportional to their velocity, which gives rise to v in the denominator. Quite non-trivial contribution $\sim v$ follows from (5). Thus, the total cross-section of the process is proportional to v^2 .

We consider now in more detail the electron captured by nucleus. The lower part of the fermion loop in Fig. 2 is described by the wave function of bound electron. For the wave function squared of this electron in the momentum representation, we confine to the non-relativistic approximation. This approximation is quite common in such problems, and usually works with a reasonable accuracy. In the present case, this wave function squared is in the momentum representation as follows [4]:

$$\frac{32\zeta^5}{\pi^2} \frac{1}{(k^2 + \zeta^2)^4}, \quad (8)$$

here and below $\zeta = Z\alpha m$.

The cross-section should be multiplied by the factor $2(Z\alpha)^4$. The factor 2 arises here since the electron can be captured either by the upper nucleus, or by the lower one; factor $(Z\alpha)^4$ corresponds to the photon exchange between two nuclei. Then, we should integrate the cross-section over the phase space of positron $\frac{d^3k}{(2\pi)^3(2k_0)}$. In this way, we arrive at the following expression for the cross-section discussed:

$$\sigma = \frac{256}{\pi} Z^4 \alpha^4 v^2 \int \frac{d^3k}{(2\pi)^3 k_0^2} \frac{\zeta^5}{(k^2 + \zeta^2)^4} = \frac{128 Z^4 \alpha^4}{\pi^3} v^2 \int_0^\infty dk \frac{\zeta^5}{(k^2 + \zeta^2)^4} \frac{k^2}{k^2 + m^2}. \quad (9)$$

The last integral in this expression is

$$\int_0^\infty dk \frac{\zeta^5}{(k^2 + \zeta^2)^4} \frac{k^2}{k^2 + m^2} = \frac{\pi}{32} \frac{1}{m^2(1 + Z\alpha)^4} (1 + 4Z\alpha + 5Z^2\alpha^2). \quad (10)$$

Thus, the cross-section of positron production in the collision of slow heavy nuclei is

$$\sigma = \frac{128}{\pi^3} Z^4 \alpha^4 v^2 \times \frac{\pi}{32} \frac{1}{m^2(1 + Z\alpha)^4} (1 + 4Z\alpha + 5Z^2\alpha^2) = \frac{4}{\pi^2} \frac{Z^4 \alpha^4}{(1 + Z\alpha)^4} (1 + 4Z\alpha + 5Z^2\alpha^2) \frac{v^2}{m^2}. \quad (11)$$

The reasonable estimate for uranium nuclei is

$$\sigma \simeq 0.07 \frac{v^2}{m^2}. \quad (12)$$

In conclusion, let us come back to the absolute value of the discussed cross-section. According to our estimate (12), it is about $3 \times 10^{-23} \text{ cm}^2$ (for $v = 0.1$). This estimate exceeds essentially the cross-section of e^+e^- production in heavy ion collisions, which is about 10^{-25} cm^2 .

Acknowledgements

We are grateful to L.N. Labzowsky and V.M. Shabaev for useful discussions.

-
- [1] U. Muller, T. de Reus, J. Reinhardt, B. Muller, W. Greiner, G. Soff, Phys. Rev. A **37**, 1449 (1988).
 - [2] I.B. Khriplovich, JETP Letters **100**, 552 (2014).
 - [3] I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn, A.I. Bondarev, Y.S. Kozhedub, G. Plunien, Th. Stohler, Phys. Rev. A **91**, 032708 (2015).
 - [4] H.A. Bethe, E.E. Salpeter, Quantum mechanics of one- and two-electron atoms, New York, Academic Press, 1957.